

PARI-GP Reference Card

(PARI-GP version 2.3.0)

Note: optional arguments are surrounded by braces {}.

Starting & Stopping GP

to enter GP, just type its name: `gp`
to exit GP, type `\q` or `quit`

Help

describe function `?function`
extended description `??keyword`
list of relevant help topics `???pattern`

Input/Output & Defaults

output previous line, the lines before `%, %', %'', etc.`
output from line n `%n`
separate multiple statements on line `;`
extend statement on additional lines `\`
extend statements on several lines `{seq1; seq2;`
comment `/* ... */`
one-line comment, rest of line ignored `\\ ...`
set default d to val `default({d},{val},flag)`
mimic behaviour of GP 1.39 `default(compatible,3)`

Metacommands

toggle timer on/off `#`
print time for last result `##`
print $%n$ in raw format `\a n`
print $%n$ in pretty format `\b n`
print defaults `\d`
set debug level to n `\g n`
set memory debug level to n `\gm n`
enable/disable logfile `\l {filename}`
print $%n$ in pretty matrix format `\m`
set output mode (raw, default, prettyprint) `\o n`
set n significant digits `\p n`
set n terms in series `\ps n`
quit GP `\q`
print the list of PARI types `\t`
print the list of user-defined functions `\u`
read file into GP `\r filename`
write $%n$ to file `\w n filename`

GP Within Emacs

to enter GP from within Emacs: `M-x gp, C-u M-x gp`
word completion `(TAB)`
help menu window `M-\c`
describe function `M-?`
display $\mathrm{T\!E\!X}$ 'd PARI manual `M-x gpman`
set prompt string `M-\p`
break line at column 100, insert `M-\\`
PARI metacommand `\letter` `M-\letter`

Reserved Variable Names

$\pi = 3.14159\dots$ `Pi`
Euler's constant $= .57721\dots$ `Euler`
square root of -1 `I`
big-oh notation `O`

PARI Types & Input Formats

| | |
|---|---|
| <code>t_INT</code> . Integers | $\pm n$ |
| <code>t_REAL</code> . Real Numbers | $\pm n.ddd$ |
| <code>t_INTMOD</code> . Integers modulo m | <code>Mod(n, m)</code> |
| <code>t_FRAC</code> . Rational Numbers | n/m |
| <code>t_COMPLEX</code> . Complex Numbers | $x + y * I$ |
| <code>t_PADIC</code> . p -adic Numbers | $x + O(p^k)$ |
| <code>t_QUAD</code> . Quadratic Numbers | $x + y * \text{quadgen}(D)$ |
| <code>t_POLMOD</code> . Polynomials modulo g | <code>Mod(f, g)</code> |
| <code>t_POL</code> . Polynomials | $a * x^n + \dots + b$ |
| <code>t_SER</code> . Power Series | $f + O(x^k)$ |
| <code>t_QFI/t_QFR</code> . Imag/Real bin. quad. forms | <code>Qfb($a, b, c, \{d\}$)</code> |
| <code>t_RFRAC</code> . Rational Functions | f/g |
| <code>t_VEC/t_COL</code> . Row/Column Vectors | $[x, y, z], [x, y, z] \sim$ |
| <code>t_MAT</code> . Matrices | $[x, y; z, t; u, v]$ |
| <code>t_LIST</code> . Lists | <code>List($[x, y, z]$)</code> |
| <code>t_STR</code> . Strings | <code>"aaa"</code> |

Standard Operators

| | |
|---------------------------------------|---|
| basic operations | <code>+, -, *, /, ^</code> |
| <code>i=i+1, i=i-1, i=i*j, ...</code> | <code>i++, i--, i*=j, ...</code> |
| euclidean quotient, remainder | $x \backslash y, x \backslash y, x \% y, \text{divrem}(x, y)$ |
| shift x left or right n bits | $x << n, x >> n$ or <code>shift(x, n)</code> |
| comparison operators | <code><=, <, >=, >, ==, !=</code> |
| boolean operators (or, and, not) | <code> , &&, !</code> |
| sign of $x = -1, 0, 1$ | <code>sign(x)</code> |
| maximum/minimum of x and y | <code>max, min(x, y)</code> |
| integer or real factorial of x | $x!$ or <code>factorial(x)</code> |
| derivative of f w.r.t. x | f' |

Conversions

Change Objects

| | |
|--|--|
| to vector, matrix, set, list, string | <code>Col/Vec, Mat, Set, List, Str</code> |
| create PARI object ($x \bmod y$) | <code>Mod(x, y)</code> |
| make x a polynomial of v | <code>Pol($x, \{v\}$)</code> |
| as above, starting with constant term | <code>Polrev($x, \{v\}$)</code> |
| make x a power series of v | <code>Ser($x, \{v\}$)</code> |
| PARI type of object x | <code>type($x, \{t\}$)</code> |
| object x with precision n | <code>prec($x, \{n\}$)</code> |
| evaluate f replacing vars by their value | <code>eval(f)</code> |

Select Pieces of an Object

| | |
|--------------------------------------|--|
| length of x | <code>#x</code> or <code>length(x)</code> |
| n -th component of x | <code>component(x, n)</code> |
| n -th component of vector/list x | $x[n]$ |
| (m, n) -th component of matrix x | $x[m, n]$ |
| row m or column n of matrix x | $x[m,], x[, n]$ |
| numerator of x | <code>numerator(x)</code> |
| lowest denominator of x | <code>denominator(x)</code> |

Conjugates and Lifts

| | |
|--|---|
| conjugate of a number x | <code>conj(x)</code> |
| conjugate vector of algebraic number x | <code>conjvec(x)</code> |
| norm of x , product with conjugate | <code>norm(x)</code> |
| square of L^2 norm of vector x | <code>norml2(x)</code> |
| lift of x from Mods | <code>lift, centerlift(x)</code> |

Random Numbers

random integer between 0 and $N - 1$ `random($\{N\}$)`
get random seed `getrand()`
set random seed to s `setrand(s)`

Lists, Sets & Sorting

sort x by k th component `vecsort($x, \{k\}, \{fl = 0\}$)`
Sets (= row vector of strings with strictly increasing entries)
intersection of sets x and y `setintersect(x, y)`
set of elements in x not belonging to y `setminus(x, y)`
union of sets x and y `setunion(x, y)`
look if y belongs to the set x `setsearch($x, y, flag$)`
Lists
create empty list of maximal length n `listcreate(n)`
delete all components of list l `listkill(l)`
append x to list l `listput($l, x, \{i\}$)`
insert x in list l at position i `listinsert(l, x, i)`
sort the list l `listsort($l, flag$)`

Programming & User Functions

Control Statements (X : formal parameter in expression seq)
eval. seq for $a \leq X \leq b$ `for($X = a, b, seq$)`
eval. seq for X dividing n `fordiv(n, X, seq)`
eval. seq for primes $a \leq X \leq b$ `forprime($X = a, b, seq$)`
eval. seq for $a \leq X \leq b$ stepping s `forstep($X = a, b, s, seq$)`
multivariable for `forvec($X = v, seq$)`
if $a \neq 0$, evaluate seq_1 , else seq_2 `if($a, \{seq_1\}, \{seq_2\}$)`
evaluate seq until $a \neq 0$ `until(a, seq)`
while $a \neq 0$, evaluate seq `while(a, seq)`
exit n innermost enclosing loops `break($\{n\}$)`
start new iteration of n th enclosing loop `next($\{n\}$)`
return x from current subroutine `return(x)`
error recovery (try seq_1) `trap($\{err\}, \{seq_2\}, \{seq_1\}$)`

Input/Output

prettyprint args with/without newline `printp(), printp1()`
print args with/without newline `print(), print1()`
read a string from keyboard `input()`
reorder priority of variables x, y, z `reorder($\{[x, y, z]\}$)`
output $args$ in $\mathrm{T\!E\!X}$ format `printtex($args$)`
write $args$ to file `write, write1, writetex($file, args$)`
read file into GP `read($\{file\}$)`

Interface with User and System

allocates a new stack of s bytes `allocatemem($\{s\}$)`
execute system command a `system(a)`
as above, feed result to GP `extern(a)`
install function from library `install($f, code, \{gpf\}, \{lib\}$)`
alias old to new `alias(new, old)`
new name of function f in GP 2.0 `whatnow(f)`

User Defined Functions

`name(formal vars) = local(local vars); seq`
`struct.member = seq`
kill value of variable or function x `kill(x)`
declare global variables `global(x, \dots)`

Iterations, Sums & Products

numerical integration `intnum($X = a, b, expr, flag$)`
sum $expr$ over divisors of n `sumdiv($n, X, expr$)`
sum $X = a$ to $X = b$, initialized at x `sum($X = a, b, expr, \{x\}$)`
sum of series $expr$ `suminf($X = a, expr$)`
sum of alternating/positive series `sumalt, sumpos`
product $a \leq X \leq b$, initialized at x `prod($X = a, b, expr, \{x\}$)`
product over primes $a \leq X \leq b$ `prodeuler($X = a, b, expr$)`
infinite product $a \leq X \leq \infty$ `prodinf($X = a, expr$)`
real root of $expr$ between a and b `solve($X = a, b, expr$)`

Vectors & Matrices

| | |
|-----------------------------------|--|
| dimensions of matrix x | <code>matsize(x)</code> |
| concatenation of x and y | <code>concat($x, \{y\}$)</code> |
| extract components of x | <code>vecextract($x, y, \{z\}$)</code> |
| transpose of vector or matrix x | <code>mattranspose(x)</code> or <code>x-</code> |
| adjoint of the matrix x | <code>matadjoin(x)</code> |
| eigenvectors of matrix x | <code>mateigen(x)</code> |
| characteristic polynomial of x | <code>charpoly($x, \{v\}, flag$)</code> |
| minimal polynomial of x | <code>minpoly($x, \{v\}$)</code> |
| trace of matrix x | <code>trace(x)</code> |

Constructors & Special Matrices

| | |
|---|--|
| row vec. of $expr$ eval'd at $1 \leq i \leq n$ | <code>vector($n, \{i\}, \{expr\}$)</code> |
| col. vec. of $expr$ eval'd at $1 \leq i \leq n$ | <code>vectorv($n, \{i\}, \{expr\}$)</code> |
| matrix $1 \leq i \leq m, 1 \leq j \leq n$ | <code>matrix($m, n, \{i\}, \{j\}, \{expr\}$)</code> |
| diagonal matrix whose diag. is x | <code>matdiagonal(x)</code> |
| $n \times n$ identity matrix | <code>matid(n)</code> |
| Hessenberg form of square matrix x | <code>mathess(x)</code> |
| $n \times n$ Hilbert matrix $H_{ij} = (i + j - 1)^{-1}$ | <code>mathilbert(n)</code> |
| $n \times n$ Pascal triangle $P_{ij} = \binom{i}{j}$ | <code>matpascal($n - 1$)</code> |
| companion matrix to polynomial x | <code>matcompanion(x)</code> |

Gaussian elimination

| | |
|--|---|
| determinant of matrix x | <code>matdet($x, flag$)</code> |
| kernel of matrix x | <code>matker($x, flag$)</code> |
| intersection of column spaces of x and y | <code>matintersect(x, y)</code> |
| solve $M * X = B$ (M invertible) | <code>matsolve(M, B)</code> |
| as solve, modulo D (col. vector) | <code>matsolvemod(M, D, B)</code> |
| one sol of $M * X = B$ | <code>matinverseimage(M, B)</code> |
| basis for image of matrix x | <code>matimage(x)</code> |
| supplement columns of x to get basis | <code>mat supplement(x)</code> |
| rows, cols to extract invertible matrix | <code>matindexrank(x)</code> |
| rank of the matrix x | <code>matrank(x)</code> |

Lattices & Quadratic Forms

| | |
|--|--|
| upper triangular Hermite Normal Form | <code>mathnf(x)</code> |
| HNF of x where d is a multiple of $\det(x)$ | <code>mathnfmod(x, d)</code> |
| elementary divisors of x | <code>matsnf(x)</code> |
| LLL-algorithm applied to columns of x | <code>qflll($x, flag$)</code> |
| like <code>qflll</code> , x is Gram matrix of lattice | <code>qflllgram($x, flag$)</code> |
| LLL-reduced basis for kernel of x | <code>matkerint(x)</code> |
| Z -lattice \longleftrightarrow Q -vector space | <code>matrixqz(x, p)</code> |
| signature of quad form $t^y * x * y$ | <code>qfsign(x)</code> |
| decomp into squares of $t^y * x * y$ | <code>qfgaussred(x)</code> |
| find up to m sols of $t^y * x * y \leq b$ | <code>qfminim(x, b, m)</code> |
| $v, v[i] :=$ number of sols of $t^y * x * y = i$ | <code>qfrep($x, B, flag$)</code> |
| eigenvals/eigenvecs for real symmetric x | <code>qfjacobi(x)</code> |

Formal & p-adic Series

| | |
|---|--|
| truncate power series or p -adic number | <code>truncate(x)</code> |
| valuation of x at p | <code>valuation(x, p)</code> |
| Dirichlet and Power Series | |
| Taylor expansion around 0 of f w.r.t. x | <code>taylor(f, x)</code> |
| $\sum a_k b_k t^k$ from $\sum a_k t^k$ and $\sum b_k t^k$ | <code>serconvol(x, y)</code> |
| $f = \sum a_k * t^k$ from $\sum (a_k / k!) * t^k$ | <code>serlaplace(f)</code> |
| reverse power series F so $F(f(x)) = x$ | <code>serreverse(f)</code> |
| Dirichlet series multiplication / division | <code>dirmul, dirdiv(x, y)</code> |
| Dirichlet Euler product (b terms) | <code>direuler($p = a, b, expr$)</code> |

p-adic Functions

| | |
|-------------------------------------|--|
| Teichmuller character of x | <code>teichmuller(x)</code> |
| Newton polygon of f for prime p | <code>newtonpoly(f, p)</code> |

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Polynomials & Rational Functions

| | |
|---|---|
| degree of f | <code>poldegree(f)</code> |
| coefficient of degree n of f | <code>polcoeff(f, n)</code> |
| round coeffs of f to nearest integer | <code>round($f, \{&e\}$)</code> |
| gcd of coefficients of f | <code>content(f)</code> |
| replace x by y in f | <code>subst(f, x, y)</code> |
| discriminant of polynomial f | <code>poldisc(f)</code> |
| resultant of f and g | <code>polresultant($f, g, flag$)</code> |
| as above, give $[u, v, d], xu + yv = d$ | <code>bezoutres(x, y)</code> |
| derivative of f w.r.t. x | <code>deriv(f, x)</code> |
| formal integral of f w.r.t. x | <code>intformal(f, x)</code> |
| reciprocal poly $x^{\deg f} f(1/x)$ | <code>polrecip(f)</code> |
| interpol. pol. eval. at a | <code>polinterpolate($X, \{Y\}, \{a\}, \{&e\}$)</code> |
| initialize t for Thue equation solver | <code>thueinit(f)</code> |
| solve Thue equation $f(x, y) = a$ | <code>thue($t, a, \{sol\}$)</code> |

Roots and Factorization

| | |
|--|--|
| number of real roots of $f, a < x \leq b$ | <code>polsturm($f, \{a\}, \{b\}$)</code> |
| complex roots of f | <code>polroots(f)</code> |
| symmetric powers of roots of f up to n | <code>polsym(f, n)</code> |
| roots of f mod p | <code>polrootsmod($f, p, flag$)</code> |
| factor f | <code>factor($f, \{lim\}$)</code> |
| factorization of f mod p | <code>factormod($f, p, flag$)</code> |
| factorization of f over \mathbf{F}_{p^a} | <code>factorff(f, p, a)</code> |
| p -adic fact. of f to prec. r | <code>factorpadic($f, p, r, flag$)</code> |
| p -adic roots of f to prec. r | <code>polrootspadic(f, p, r)</code> |
| p -adic root of f cong. to a mod p | <code>padicappr(f, a)</code> |
| Newton polygon of f for prime p | <code>newtonpoly(f, p)</code> |

Special Polynomials

| | |
|--|--|
| n th cyclotomic polynomial in var. v | <code>polcyclo($n, \{v\}$)</code> |
| d -th degree subfield of $\mathbf{Q}(\zeta_n)$ | <code>polsubcyclo($n, d, \{v\}$)</code> |
| n -th Legendre polynomial | <code>pollegendre(n)</code> |
| n -th Tchebicheff polynomial | <code>poltchebi(n)</code> |
| Zagier's polynomial of index n, m | <code>polzagier(n, m)</code> |

Transcendental Functions

| | |
|--|---|
| real, imaginary part of x | <code>real(x), imag(x)</code> |
| absolute value, argument of x | <code>abs(x), arg(x)</code> |
| square/ n th root of x | <code>sqrtn($x, n, &z$)</code> |
| trig functions | <code>sin, cos, tan, cotan</code> |
| inverse trig functions | <code>asin, acos, atan</code> |
| hyperbolic functions | <code>sinh, cosh, tanh</code> |
| inverse hyperbolic functions | <code>asinh, acosh, atanh</code> |
| exponential of x | <code>exp(x)</code> |
| natural log of x | <code>ln(x)</code> or <code>log(x)</code> |
| gamma function $\Gamma(x) = \int_0^\infty e^{-t} t^{x-1} dt$ | <code>gamma(x)</code> |
| logarithm of gamma function | <code>lngamma(x)</code> |
| $\psi(x) = \Gamma'(x) / \Gamma(x)$ | <code>psi(x)</code> |
| incomplete gamma function ($y = \Gamma(s)$) | <code>incgam($s, x, \{y\}$)</code> |
| exponential integral $\int_x^\infty e^{-t} / t dt$ | <code>eint1(x)</code> |
| error function $2 / \sqrt{\pi} \int_x^\infty e^{-t^2} dt$ | <code>erfc(x)</code> |
| dilogarithm of x | <code>dilog(x)</code> |
| m th polylogarithm of x | <code>polylog($m, x, flag$)</code> |
| U -confluent hypergeometric function | <code>hyperu(a, b, u)</code> |
| J -Bessel function $J_{n+1/2}(x)$ | <code>besseljh(n, x)</code> |
| K -Bessel function of index nu | <code>besselk(nu, x)</code> |

Elementary Arithmetic Functions

| | |
|--|---|
| vector of binary digits of $ x $ | <code>binary(x)</code> |
| give bit number n of integer x | <code>bittest(x, n)</code> |
| ceiling of x | <code>ceil(x)</code> |
| floor of x | <code>floor(x)</code> |
| fractional part of x | <code>frac(x)</code> |
| round x to nearest integer | <code>round($x, \{&e\}$)</code> |
| truncate x | <code>truncate($x, \{&e\}$)</code> |
| gcd/LCM of x and y | <code>gcd(x, y), lcm(x, y)</code> |
| gcd of entries of a vector/matrix | <code>content(x)</code> |
| Primes and Factorization | |
| add primes in v to the prime table | <code>addprimes(v)</code> |
| the n th prime | <code>prime(n)</code> |
| vector of first n primes | <code>primes(n)</code> |
| smallest prime $\geq x$ | <code>nextprime(x)</code> |
| largest prime $\leq x$ | <code>precprime(x)</code> |
| factorization of x | <code>factor($x, \{lim\}$)</code> |
| reconstruct x from its factorization | <code>factorback($fa, \{nf\}$)</code> |

Divisors

| | |
|---|---|
| number of distinct prime divisors | <code>omega(x)</code> |
| number of prime divisors with mult | <code>bigomega(x)</code> |
| number of divisors of x | <code>numdiv(x)</code> |
| row vector of divisors of x | <code>divisors(x)</code> |
| sum of (k -th powers of) divisors of x | <code>sigma($x, \{k\}$)</code> |

Special Functions and Numbers

| | |
|--|--|
| binomial coefficient $\binom{x}{y}$ | <code>binomial(x, y)</code> |
| Bernoulli number B_n as real | <code>bernreal(n)</code> |
| Bernoulli vector B_0, B_2, \dots, B_{2n} | <code>bernvec(n)</code> |
| n th Fibonacci number | <code>fibonacci(n)</code> |
| number of partitions of n | <code>numbpart(n)</code> |
| Euler ϕ -function | <code>eulerphi(x)</code> |
| Möbius μ -function | <code>moebius(x)</code> |
| Hilbert symbol of x and y (at p) | <code>hilbert($x, y, \{p\}$)</code> |
| Kronecker-Legendre symbol $(\frac{x}{y})$ | <code>kronecker(x, y)</code> |

Miscellaneous

| | |
|--|--|
| integer or real factorial of x | <code>x!</code> or <code>fact(x)</code> |
| integer square root of x | <code>sqrntint(x)</code> |
| solve $z \equiv x$ and $z \equiv y$ | <code>chinese(x, y)</code> |
| minimal u, v so $xu + yv = \gcd(x, y)$ | <code>bezout(x, y)</code> |
| multiplicative order of x (intmod) (i=0) | <code>znorder($x, \{o\}$)</code> |
| primitive root mod prime power q | <code>znprimroot(q)</code> |
| structure of $(\mathbf{Z}/n\mathbf{Z})^*$ | <code>znstar(n)</code> |
| continued fraction of x | <code>contfrac($x, \{b\}, \{lmax\}$)</code> |
| last convergent of continued fraction x | <code>contfracpnqn(x)</code> |
| best rational approximation to x | <code>bestappr(x, k)</code> |

True-False Tests

| | |
|--|---|
| is x the disc. of a quadratic field? | <code>isfundamental(x)</code> |
| is x a prime? | <code>isprime(x)</code> |
| is x a strong pseudo-prime? | <code>ispseudoprime(x)</code> |
| is x square-free? | <code>issquarefree(x)</code> |
| is x a square? | <code>Z_issquare($x, \{&n\}$)</code> |
| is pol irreducible? | <code>polisirreducible(pol)</code> |

Based on an earlier version by Joseph H. Silverman

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PARI-GP Reference Card (2)

(PARI-GP version 2.3.0)

Elliptic Curves

Elliptic curve initially given by 5-tuple $E = [a_1, a_2, a_3, a_4, a_6]$. Points are $[x, y]$, the origin is $[0]$.

Initialize elliptic struct. ell , i.e create `ellinit($E, flag$)`

$a_1, a_2, a_3, a_4, a_6, b_2, b_4, b_6, b_8, c_4, c_6, disc, j$. This data can be recovered by typing $ell.a1, \dots, ell.j$. If fl omitted, also

E defined over **R**

| | |
|-----------------------------------|------------------------|
| x -coords. of points of order 2 | <code>ell.roots</code> |
| real and complex periods | <code>ell.omega</code> |
| associated quasi-periods | <code>ell.eta</code> |
| volume of complex lattice | <code>ell.area</code> |

E defined over \mathbf{Q}_p , $|j|_p > 1$

| | |
|-------------------------------------|------------------------|
| x -coord. of unit 2 torsion point | <code>ell.roots</code> |
| Tate's $[u^2, u, q]$ | <code>ell.tate</code> |
| Mestre's w | <code>ell.w</code> |

change curve E using $v = [u, r, s, t]$ `ellchangecurve(ell, v)`

change point z using $v = [u, r, s, t]$ `ellchangepoint(z, v)`

cond, min mod, Tamagawa num $[N, v, c]$ `ellglobalred(ell)`

Kodaira type of p fiber of E `elllocalred(ell, p)`

add points $z_1 + z_2$ `elladd(ell, z_1, z_2)`

subtract points $z_1 - z_2$ `ellsub(ell, z_1, z_2)`

compute $n \cdot z$ `ellpow(ell, z, n)`

check if z is on E `ellisoncurve(ell, z)`

order of torsion point z `ellorder(ell, z)`

torsion subgroup with generators `elltors(ell)`

y -coordinates of point(s) for x `ellordinate(ell, x)`

canonical bilinear form taken at z_1, z_2 `ellbil(ell, z_1, z_2)`

canonical height of z `ellheight($ell, z, flag$)`

height regulator matrix for pts in x `ellheightmatrix(ell, x)`

p th coeff a_p of L -function, p prime `ellap(ell, p)`

k th coeff a_k of L -function `ellak(ell, k)`

vector of first n a_k 's in L -function `ellan(ell, n)`

$L(E, s)$, set $A \approx 1$ `elllseries($ell, s, \{A\}$)`

root number for $L(E, \cdot)$ at p `ellrootno($ell, \{p\}$)`

modular parametrization of E `elltaniyama(ell)`

point $[\wp(z), \wp'(z)]$ corresp. to z `ellztopoint(ell, z)`

complex z such that $p = [\wp(z), \wp'(z)]$ `ellpointtoz(ell, p)`

Elliptic & Modular Functions

arithmetic-geometric mean `agm(x, y)`

elliptic j -function $1/q + 744 + \dots$ `ellj(x)`

Weierstrass σ function `ellsigma($ell, z, flag$)`

Weierstrass \wp function `ellwp($ell, \{z\}, flag$)`

Weierstrass ζ function `ellzeta(ell, z)`

modified Dedekind η func. $\prod(1 - q^n)$ `eta($x, flag$)`

Jacobi sine theta function `theta(q, z)`

k-th derivative at $z=0$ of `thetanullk(q, k)`

Weber's f functions `weber($x, flag$)`

Riemann's zeta $\zeta(s) = \sum n^{-s}$ `zeta(s)`

Graphic Functions

crude graph of $expr$ between a and b `plot($X = a, b, expr$)`

High-resolution plot (immediate plot)

plot $expr$ between a and b `plotth($X = a, b, expr, flag, \{n\}$)`

plot points given by lists lx, ly `plotthraw($lx, ly, flag$)`

terminal dimensions `plotsizes()`

Rectwindow functions

init window w , with size x, y `plotinit(w, x, y)`

erase window w `plotkill(w)`

copy w to w_2 with offset (dx, dy) `plotcopy(w, w_2, dx, dy)`

scale coordinates in w `plotscale(w, x_1, x_2, y_1, y_2)`

`plotth` in w `plotrecth($w, X = a, b, expr, flag, \{n\}$)`

`plotthraw` in w `plotrecthraw($w, data, flag$)`

draw window w_1 at $(x_1, y_1), \dots$ `plotdraw($[[w_1, x_1, y_1], \dots]$)`

Low-level Rectwindow Functions

set current drawing color in w to c `plotcolor(w, c)`

current position of cursor in w `plotcursor(w)`

write s at cursor's position `plotstring(w, s)`

move cursor to (x, y) `plotmove(w, x, y)`

move cursor to $(x + dx, y + dy)$ `plotrmove(w, dx, dy)`

draw a box to (x_2, y_2) `plotbox(w, x_2, y_2)`

draw a box to $(x + dx, y + dy)$ `plotrbox(w, dx, dy)`

draw polygon `plotlines($w, lx, ly, flag$)`

draw points `plotpoints(w, lx, ly)`

draw line to $(x + dx, y + dy)$ `plotrline(w, dx, dy)`

draw point $(x + dx, y + dy)$ `plotrpoint(w, dx, dy)`

Postscript Functions

as `plotth` `psplotth($X = a, b, expr, flag, \{n\}$)`

as `plotthraw` `psplotthraw($lx, ly, flag$)`

as `plotdraw` `psdraw($[[w_1, x_1, y_1], \dots]$)`

Binary Quadratic Forms

create $ax^2 + bxy + cy^2$ (distance d) `qfb($a, b, c, \{d\}$)`

reduce x ($s = \sqrt{D}$, $l = \lfloor s \rfloor$) `qfbred($x, flag, \{D\}, \{l\}, \{s\}$)`

composition of forms $x*y$ or `qfbnucomp(x, y, l)`

n -th power of form x^n or `qfbnupow(x, n)`

composition without reduction `qfbcompraw(x, y)`

n -th power without reduction `qfbpowraw(x, n)`

prime form of disc. x above prime p `qfbprimeform(x, p)`

class number of disc. x `qfbclassno(x)`

Hurwitz class number of disc. x `qfbhclassno(x)`

Quadratic Fields

quadratic number $\omega = \sqrt{x}$ or $(1 + \sqrt{x})/2$ `quadgen(x)`

minimal polynomial of ω `quadpoly(x)`

discriminant of $\mathbf{Q}(\sqrt{D})$ `quaddisc(x)`

regulator of real quadratic field `quadregulator(x)`

fundamental unit in real $\mathbf{Q}(x)$ `quadunit(x)`

class group of $\mathbf{Q}(\sqrt{D})$ `quadclassunit($D, flag, \{t\}$)`

Hilbert class field of $\mathbf{Q}(\sqrt{D})$ `quadhilbert($D, flag$)`

ray class field modulo f of $\mathbf{Q}(\sqrt{D})$ `quadrday($D, f, flag$)`

General Number Fields: Initializations

A number field K is given by a monic irreducible $f \in \mathbf{Z}[X]$.

init number field structure nf `nfinit($f, flag$)`

nf members:

| | |
|--|---------------------------|
| polynomial defining nf , $f(\theta) = 0$ | <code>nf.pol</code> |
| number of real/complex places | <code>nf.r1, nf.r2</code> |
| discriminant of nf | <code>nf.disc</code> |
| T_2 matrix | <code>nf.t2</code> |
| vector of roots of f | <code>nf.roots</code> |
| integral basis of \mathbf{Z}_K as powers of θ | <code>nf.zk</code> |
| different | <code>nf.diff</code> |
| codifferent | <code>nf.codiff</code> |

recompute nf using current precision `nfnewprec(nf)`

init relative rmf given by $g = 0$ over K `rmfinit(nf, g)`

init bnf structure `bnfinit($f, flag$)`

bnf members: same as nf , plus

| | |
|-------------------|-----------------------|
| underlying nf | <code>bnf.nf</code> |
| classgroup | <code>bnf.clgp</code> |
| regulator | <code>bnf.reg</code> |
| fundamental units | <code>bnf.fu</code> |
| torsion units | <code>bnf.tu</code> |
| $[tu, fu]$ | <code>bnf.tufu</code> |

compute a bnf from small bnf `bnfmake($sbnf$)`

add S -class group and units, yield bnf s `bnfsunit(nf, S)`

init class field structure bnr `bnrinit($bnf, m, flag$)`

bnr members: same as bnf , plus

| | |
|-----------------------------------|-----------------------|
| underlying bnf | <code>bnr.bnf</code> |
| structure of $(\mathbf{Z}_K/m)^*$ | <code>bnr.zkst</code> |

Simple Arithmetic Invariants (nf)

Elements are rational numbers, polynomials, polmods, or column vectors (on integral basis $nf.zk$).

integral basis of field def. by $f = 0$ **nfbasis**(f)
field discriminant of field $f = 0$ **nfdisc**(f)
reverse polmod $a = A(X) \bmod T(X)$ **modreverse**(a)
Galois group of field $f = 0$, $\deg f \leq 11$ **polgalois**(f)
smallest poly defining $f = 0$ **polredabs**($f, flag$)
small polys defining subfields of $f = 0$ **polred**($f, flag, \{p\}$)
small polys defining suborders of $f = 0$ **polredord**(f)
poly of degree $\leq k$ with root $x \in \mathbf{C}$ **algdep**(x, k)
small linear rel. on coords of vector x **lindep**(x)
are fields $f = 0$ and $g = 0$ isomorphic? **nfisom**(f, g)
is field $f = 0$ a subfield of $g = 0$? **nfisincl**(f, g)
compositum of $f = 0$, $g = 0$ **polcompositum**($f, g, flag$)
basic element operations (prefix **nfelt**):

(**nfelt**)**mul**, **pow**, **div**, **diveuc**, **mod**, **divrem**, **val**
express x on integer basis **nfalgtobasis**(nf, x)
express element x as a polmod **nfbasistoalg**(nf, x)
quadratic Hilbert symbol (at p) **nfhilbert**($nf, a, b, \{p\}$)
roots of g belonging to nf **nfroots**($\{nf\}, g$)
factor g in nf **nfactor**(nf, g)
factor g mod prime pr in nf **nfactormod**(nf, g, pr)
number of roots of unity in nf **nfrootsof1**(nf)
conjugates of a root θ of nf **nfgaloisconj**($nf, flag$)
apply Galois automorphism s to x **nfgaloisapply**(nf, s, x)
subfields (of degree d) of nf **nfsubfields**($nf, \{d\}$)

Dedekind Zeta Function ζ_K

ζ_K as Dirichlet series, $N(I) < b$ **dirzetak**(nf, b)
init nfz for field $f = 0$ **zetakinit**(f)
compute $\zeta_K(s)$ **zetak**($nfz, s, flag$)
Artin root number of K **bnrrootnumber**($bnr, chi, flag$)

Class Groups & Units (bnf, bnr)

$a_1, \{a_2\}, \{a_3\}$ usually $bnr, subgp$ or $bnf, module, \{subgp\}$
remove GRH assumption from bnf **bnfcertify**(bnf)
expo. of ideal x on class gp **bnfisprincipal**($bnf, x, flag$)
expo. of ideal x on ray class gp **bnrisprincipal**($bnr, x, flag$)
expo. of x on fund. units **bnfisunit**(bnf, x)
as above for S -units **bnfissunit**($bnfs, x$)
fundamental units of bnf **bnfunit**(bnf)
signs of real embeddings of $bnf.fu$ **bnfsignunit**(bnf)

Class Field Theory

ray class group structure for mod. m **bnrclass**($bnf, m, flag$)
ray class number for mod. m **bnrclassno**(bnf, m)
discriminant of class field ext **bnrdisc**($a_1, \{a_2\}, \{a_3\}$)
ray class numbers, l list of mods **bnrclassnolist**(bnf, l)
discriminants of class fields **bnrdisclist**($bnf, l, \{arch\}, flag$)
decode output from **bnrdisc** **bnfdecodemodule**(nf, fa)
is modulus the conductor? **bnrisconductor**($a_1, \{a_2\}, \{a_3\}$)
conductor of character chi **bnrconductorofchar**(bnr, chi)
conductor of extension **bnrconductor**($a_1, \{a_2\}, \{a_3\}, flag$)
conductor of extension def. by g **rnfconductor**(bnf, g)
Artin group of ext. def'd by g **rnfnormgroup**(bnr, g)
subgroups of bnr , index $\leq b$ **subgrouplist**($bnr, b, flag$)
rel. eq. for class field def'd by sub **rnfkummer**($bnr, sub, \{d\}$)
same, using Stark units (real field) **bnrstark**($bnr, sub, flag$)

PARI-GP Reference Card (2)

(PARI-GP version 2.3.0)

Ideals

Ideals are elements, primes, or matrix of generators in HNF.
is id an ideal in nf ? **nfisideal**(nf, id)
is x principal in bnf ? **bnfisprincipal**(bnf, x)
principal ideal generated by x **idealprincipal**(nf, x)
principal idele generated by x **ideleprincipal**(nf, x)
give $[a, b]$, s.t. $a\mathbf{Z}_K + b\mathbf{Z}_K = x$ **idealtwoelt**($nf, x, \{a\}$)
put ideal a ($a\mathbf{Z}_K + b\mathbf{Z}_K$) in HNF form **idealhnf**($nf, a, \{b\}$)
norm of ideal x **idealnrm**(nf, x)
minimum of ideal x (direction v) **idealmin**(nf, x, v)
LLL-reduce the ideal x (direction v) **idealred**($nf, x, \{v\}$)

Ideal Operations

add ideals x and y **idealadd**(nf, x, y)
multiply ideals x and y **idealmul**($nf, x, y, flag$)
intersection of ideals x and y **idealintersect**($nf, x, y, flag$)
 n -th power of ideal x **idealpow**($nf, x, n, flag$)
inverse of ideal x **idealinv**(nf, x)
divide ideal x by y **idealdiv**($nf, x, y, flag$)
Find $(a, b) \in x \times y$, $a + b = 1$ **idealaddtoone**($nf, x, \{y\}$)

Primes and Multiplicative Structure

factor ideal x in nf **idealfactor**(nf, x)
recover x from its factorization in nf **factorback**(x, nf)
decomposition of prime p in nf **idealprimedec**(nf, p)
valuation of x at prime ideal pr **idealval**(nf, x, pr)
weak approximation theorem in nf **idealchinese**(nf, x, y)
give bid = structure of $(\mathbf{Z}_K/id)^*$ **idealstar**($nf, id, flag$)
discrete log of x in $(\mathbf{Z}_K/bid)^*$ **ideallog**(nf, x, bid)
idealstar of all ideals of norm $\leq b$ **ideallist**($nf, b, flag$)
add archimedean places **ideallistarch**($nf, b, \{ar\}, flag$)
init **prmod** structure **nfmodprinit**(nf, pr)
kernel of matrix M in $(\mathbf{Z}_K/pr)^*$ **nfkermodpr**($nf, M, prmod$)
solve $Mx = B$ in $(\mathbf{Z}_K/pr)^*$ **nfsolvemodpr**($nf, M, B, prmod$)

Galois theory over \mathbf{q}

initializes a Galois group structure **galoisinit**($pol, \{den\}$)
action of p in **nfgaloisconj** form **galoispermopol**($G, \{p\}$)
identifies as abstract group **galoisidentify**(G)
exports a group for GAP or MAGMA **galoisexport**($G, flag$)
subgroups of the Galois group G **galoissubgroups**(G)
subfields from subgroups of G **galoissubfields**($G, flag, \{v\}$)
fixed field **galoisfixedfield**($G, perm, flag, \{v\}$)
is G abelian? **galoisisabelian**($G, flag$)
abelian number fields **galoissubcyclo**($N, H, flag, \{v\}$)

Relative Number Fields (rnf)

Extension L/K is defined by $g \in K[x]$. We have $order \subset L$.
absolute equation of L **rnfequation**($nf, g, flag$)
relative **nfalgtobasis** **rnfalgtobasis**(rnf, x)
relative **nfbasistoalg** **rnfbasistoalg**(rnf, x)
relative **idealhnf** **rnfidealhnf**(rnf, x)
relative **idealmul** **rnfidealmul**(rnf, x, y)
relative **idealtwoelt** **rnfidealtwoelt**(rnf, x)

Lifts and Push-downs

absolute \rightarrow relative repres. for x **rnfeltabstorel**(rnf, x)
relative \rightarrow absolute repres. for x **rnfeltreltoabs**(rnf, x)
lift x to the relative field **rnfeltup**(rnf, x)
push x down to the base field **rnfeltdown**(rnf, x)
idem for x ideal: (**rnfideal**)**reltoabs**, **abstorel**, **up**, **down**

Projective \mathbf{Z}_K -modules, maximal order

relative **polred** **rnfpolred**(nf, g)
relative **polredabs** **rnfpolredabs**(nf, g)
characteristic poly. of $a \bmod g$ **rnfcharpoly**($nf, g, a, \{v\}$)
relative Dedekind criterion, prime pr **rnfdedekind**(nf, g, pr)
discriminant of relative extension **rnfdisc**(nf, g)
pseudo-basis of \mathbf{Z}_L **rnfpseudobasis**(nf, g)
relative HNF basis of $order$ **rnfhnfbasis**($bnf, order$)
reduced basis for $order$ **rnflllgram**($nf, g, order$)
determinant of pseudo-matrix A **rnfdet**(nf, A)
Steinitz class of $order$ **rnfstesinitz**($nf, order$)
is $order$ a free \mathbf{Z}_K -module? **rnfisfree**($bnf, order$)
true basis of $order$, if it is free **rnfbasis**($bnf, order$)

Norms

absolute norm of ideal x **rnfidealnrmabs**(rnf, x)
relative norm of ideal x **rnfidealnrmrel**(rnf, x)
solutions of $N_{K/\mathbf{Q}}(y) = x \in \mathbf{Z}$ **bnfisintnorm**(bnf, x)
is $x \in \mathbf{Q}$ a norm from K ? **bnfisnorm**($bnf, x, flag$)
initialize T for norm eq. solver **rnfisnorminit**($K, pol, flag$)
is $a \in K$ a norm from L ? **rnfisnorm**($T, a, flag$)

Based on an earlier version by Joseph H. Silverman

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